Spring Mass Characteristics of the Fastest Men on Earth

Abstract

The spring mass model has widely been used to characterize the whole body during running and sprinting. However the spring mass characteristics of the world’s fastest men are still unknown. Thus the aim of this study was to model these characteristics for currently the 3 fastest men on earth (Usain Bolt, Tyson Gay and Asafa Powell). This was done by using data collected during the 2009 World championships in Berlin and the modelling method of Morin et al. [21]. Even though Bolt achieved the greatest velocity \(12.3 \text{ m.s}^{-1}\) over the 60–80m split compared to his competitors, his estimated vertical stiffness \(355.8 \text{ kN.m}^{-1}\) and leg stiffness \(21.0 \text{ kN.m}^{-1}\) were significantly lower than his competitors. This reduction in stiffness is a consequence of Bolt’s longer contact time (0.091 s) and lower step frequency (4.49Hz). Thus Bolt is able to run at a greater velocity but with lower stiffness compared to his competitors.

Introduction

The 100m world record, currently held by Usain Bolt (UB), is 9.58 s. UB is clearly a phenomenal athlete and our work has suggested that his stature and his reduced step frequency facilitate his success resulting in an advantage in relative power development and mechanical efficiency compared to his competitors [3]. Furthermore our past work [4] supported the concept [3,25] that the longer steps of UB with longer ground contact times and longer distances travelled during ground contact generated higher impulses resulting in an exceptionally fast winning time. However we do not know what the spring mass characteristics of the world’s fastest men are. Leg and vertical stiffness is often cited as increasing as speed increases, but these studies have been at relatively low speeds \((-8 \text{ m.s}^{-1})\) [1,6,14,21] and not the speeds of world class sprinters.

The spring-mass model (SMM) is used to model both the vertical motion of the centre of mass (CoM) during contact and the stiffness of the leg spring. It has widely been used to characterize the whole body during running and sprinting. The calculation of the effective vertical stiffness \((K_v)\) is derived from the maximum vertical force and the displacement of the CoM \((\Delta y_c)\). During running the leg sweeps through an angle thus it is not directly over the CoM (as modelled for \(K_v\)) therefore leg stiffness \((K_{leg})\) is calculated. These variables are usually derived from double integration of force data. However, a method proposed by Morin et al. [21] mathematically models the force curve as a sine-wave using mass, stature, flight \((t_f)\) and contact \((t_c)\) times, and velocity data. This model has allowed the SMM to be calculated during the 100 m for novice sprinters \((t_{100} 14.21 \text{ s}; \bar{\upsilon} 7.06 \text{ m.s}^{-1})\), and the 400 m of ‘well trained athletes’ \((t_{400} 52.67 \text{ s})\) whilst running on the track [15,22]. An intriguing application of this approach is to use it in elite competitive events where only running speed and step frequency data are available for \(20 \text{ m} \) splits. It was therefore the aim of this study to adopt the model of Morin et al. [21] and apply it to the top 3 male sprinters in the 100m World Athletics Championship final of 2009. It was hypothesized that even though UB runs at a greater velocity than his fellow competitors his leg and vertical stiffness would be less due to his reduced step frequency [11] and the requirement for increased impulse [4].

Method

This study was performed in accordance with the ethical standards laid out by the IJSM [12]. The IAAF commissioned a biomechanics project,
undertaken by the German Athletic Federation, which produced individual split times and corresponding step rates for each athlete during the 100 m final at the 2009 Athletics World Championships in Berlin [12]. We have used this data in previous studies [3,4] to calculate physiological and biomechanical parameters for the first 3 finishers (UB; Tyson Gay, TG; Asafa Powell, AP). We therefore choose to estimate the SMM characteristics for these same athletes. These athletes were chosen for analysis because to date they are the 3 fastest sprinters of all time (100 m personal bests: UB 9.58 s, TG 9.69 s, and AP 9.72 s) who between them hold the top 18 all-time 100 m times [29]. Anthropometric data for all 3 sprinters (Table 1) were gathered from available reference sources [27]. UB’s body mass was adjusted based on a personal statement about his stature and on and off season body mass [9,28].

The velocity profile (Fig. 1) of each sprinter was modelled based on the distance covered after individual split times using the integral of a bi-exponential model for UB (black line), TG (light grey line) and AP (dark grey line).

Fig. 1 Velocity profile derived from the bi-exponential model for UB (black line), TG (light grey line) and AP (dark grey line).

Statistical analysis

Descriptive statistics (means and standard deviations) and a non-parametric Kruskal-Wallis-Test with a post-hoc Mann-Whitney-Test were used to determine changes in spring mass variables between the 3 sprinters. Statistical significant level was set at p<0.05. Statistical analysis was performed on SPSS (v 16.0).

Results

The average velocity achieved by UB, TG, and AP over the 60–80 m splits was 12.19 (0.26) m s⁻¹. Step frequency was markedly less for UB compared to TG and AP (Table 2). Estimated F max was significantly greater for UB compared to AP and TG, and AP was significantly greater than TG (Table 2). Estimated K vert could be calculated. Leg stiffness was derived from the maximum vertical force and the change in leg length (eq. 4).

\[ K_{\text{vert}} = F_{\text{max}} \cdot \Delta y_c^{-1} \]  (1)

\[ F_{\text{max}} = \frac{\text{mass} \cdot g \cdot \frac{1}{2} \left( \frac{t_f}{t_c} + 1 \right)}{2} \]  (2)

where g is the gravitational acceleration. \( \Delta y_c \) (in meters) is the maximal downward displacement of the CoM during contact and was calculated using equation 3.

\[ \Delta y_c = \frac{F_{\text{max}} \cdot \frac{t_f^2}{2} + g \cdot \frac{t_f^2}{8}}{\text{mass}} \]  (3)

\[ K_{\text{leg}} = F_{\text{max}} \cdot \Delta L^{-1} \]  (4)

where L is leg length (in metres) and is modelled from each athlete’s stature according to Winter [26]. Leg length obtained this way has no significant effect on the stiffness values obtained using the method of Morin et al. [21].

\[ L = 0.53h \]  (5)

The change in leg length (\( \Delta L \), in meters) was calculated from

\[ \Delta L = L - \sqrt{L^2 - \left( \frac{v_{100}}{2g} \right)^2 + \Delta y_c} \]  (6)

Table 1 Anthropometric and performance data from the 2009 World Championship.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>UB</th>
<th>TG</th>
<th>AP</th>
</tr>
</thead>
<tbody>
<tr>
<td>age (yrs)</td>
<td>24</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>stature (m)</td>
<td>1.96</td>
<td>1.83</td>
<td>1.90</td>
</tr>
<tr>
<td>body mass (kg)</td>
<td>95</td>
<td>73</td>
<td>88</td>
</tr>
<tr>
<td>t100 (s)</td>
<td>9.58</td>
<td>9.71</td>
<td>9.84</td>
</tr>
<tr>
<td>( v_{100} ) (m s⁻¹)</td>
<td>10.44</td>
<td>10.30</td>
<td>10.16</td>
</tr>
</tbody>
</table>

Table 2 Estimated SMM characteristics for 60–80 m.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>UB m</th>
<th>TG m</th>
<th>AP m</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_c ) (s)</td>
<td>0.091±0.001*#</td>
<td>0.070±0.001*</td>
<td>0.080±0.001</td>
</tr>
<tr>
<td>( v_c ) (m s⁻¹)</td>
<td>12.3±0.02*</td>
<td>12.1±0.02*</td>
<td>11.9±0.01</td>
</tr>
<tr>
<td>( F_{\text{max}} ) (kN)</td>
<td>3.60±0.001*#</td>
<td>3.25±0.002*</td>
<td>3.59±0.001</td>
</tr>
<tr>
<td>( K_{\text{vert}} ) (kN m⁻¹)</td>
<td>535.8±0.46*</td>
<td>541.8±0.89*</td>
<td>457.0±0.47</td>
</tr>
<tr>
<td>( K_{\text{leg}} ) (kN m⁻¹)</td>
<td>21.0±0.05*#</td>
<td>31.0±0.05*</td>
<td>28.4±0.05</td>
</tr>
<tr>
<td>( \Delta y_c ) (m)</td>
<td>0.01±0.0001*#</td>
<td>0.006±0.0001*</td>
<td>0.008±0.0001</td>
</tr>
<tr>
<td>( \Delta ) (m)</td>
<td>0.17±0.0001*</td>
<td>0.10±0.0001*</td>
<td>0.13±0.0001</td>
</tr>
<tr>
<td>SF (Hz)</td>
<td>4.49</td>
<td>4.96</td>
<td>4.74</td>
</tr>
</tbody>
</table>

All data significant to * p < 0.01; # Significant difference to AP; § significant difference to TG. ± standard deviation
and $K_{leg}$ for UB were significantly less compared to TG and AP. Estimated $K_{vert}$ for UB was 52% and 28% less than TG and AP respectively and estimated $K_{leg}$ was 47% and 35% less compared to TG and AP, respectively. TG had the greatest stiffness characteristics of all the sprinters and this was in part due to the significantly shorter $t_t$ and reduced $\Delta Y_t$ and $\Delta L$ compared to UB and AP (\(\text{\textbullet Table 2}\)). $t_t$ was statistically significantly different between all athletes, however this was due to the calculation method of flight time and it is in practical terms not a significant finding.

**Discussion**

The spring mass characteristics of world class sprinters, in competition, who run at velocities in the region of 12.19 (0.26) m$^{-1}$ have not previously been reported. Morin et al. [21] reported the spring mass characteristics of novice sprinters running at a velocity of 8.24 (0.24) m$^{-1}$ (during 60–80m split), the sprinters of Arampatzis et al. [1] ran at 6.59 (0.24) m$^{-1}$, while the sprinters of He et al. [14] and Cavagna et al. [8] ran at 6 m$^{-1}$ and 5 m$^{-1}$, respectively.

Prior to discussing the results it should be made clear that these data are modelled and are therefore estimates. We acknowledge that there may be limitations in our approach, but the opportunity to take direct measurements in a world championship final to calculate SMM characteristics is, at the moment, unlikely to happen. The sine-wave model of calculating stiffness [21] has been shown to be a valid method of estimating stiffness. However it does have limitations. Namely the model assumes a constant point of force application on the ground during stance phase, when the location actually moves 0.16 m [17]. The limitations associated with spring-mass models per se are also applicable to the sine-wave model [21].

The sine-wave model requires flight ($t_f$) and contact time ($t_c$) which were not directly measured during the 100m final. We estimated $t_t$ and $t_c$ via step frequency, which allows the calculation of step time. $t_t$ data were derived from Weyand et al. [25] which along with step time allowed $t_c$ be to calculated. Both $t_t$ and $t_c$ were comparable to those measured via kinematics, accelerometry or on an instrumented treadmill [10,15,22]. Furthermore our data was also comparable to world class sprinters (Ben Johnson (11.76 m$^{-1}$)/Carl Lewis (11.63 m$^{-1}$)) recorded at the 1987 World Athletic championships with $t_t$ of 0.082/0.085 s and a $t_c$ of 0.122/0.138 s at the 60m split for both sprinters [20]. Therefore the calculated $t_t$ and $t_c$ data can be used in this context to estimate SMM. Lastly, we were unable to take direct measurements of mass, stature and leg length. Therefore we used anthropometric data freely available in the public domain. These data have been used in previous sprinting, and sprinting and anthropometric studies [3,9]. Anthropometrical measures have a 1:1 weighting (or less) on stiffness measures [21]. A percentage change in leg length has virtually no impact on vertical or leg stiffness whereas a 10% reduction in body mass has a 10% reduction in stiffness and vice versa [21].

Running velocity is the product of step frequency (SF) and step length (SL). An inverse relationship exists between SF and SL at maximum effort, thus an increase in SF for example will lead to a decrease in SL and vice versa. UB had a longer step length to accompany his slower step frequency whereas TG and AP had shorter step lengths to accompany their faster step frequency [12]. To accommodate the higher step frequencies during running the leg spring becomes stiffer [11]. The size of the sprinter will impact upon these spatial parameters which in turn will impact upon the stiffness characteristics. We only simulated the spring mass characteristics for 3 sprinters with UB perhaps exhibiting an extreme in morphology compared to his competitors and that there may be greater variance in body size among elite sprinters. However, Charles and Bejan stated that world record holders in the 100m sprint are becoming taller and heavier [9]. This agreed with Watts et al. [24] who studied not just the world record holders but the top 10 athletes spanning 10 decades of recorded competition. They found that the reciprocal ponderal index (indicating that athletes have become taller and more linear) is a more significant factor of success. Bejan et al. further explored the evolution of height and sprinting speed and suggested that it is the height which the CoM falls from which is indicative of sprinting performance – if the CoM falls from a greater height the more advantageous [5]. The location of the CoM is dependent upon the morphology of the body, thus an athlete with longer limbs and narrower circumferences of body segments (i.e., the shanks) will result in a higher position of the CoM. The results support that UB's tall stature enabled for longer steps with longer ground contact times and longer distances travelled during ground contact [4]. This required lower force and power to generate higher impulses during ground contact under favourable conditions of force generation [3] and biomechanical efficiency [3].

Estimated $F_{max}$ was greater than that reported for slower sprinters [i.e., 22] and comparable to sprinters running at 10.37 m$^{-1}$ [6]. The greater $F_{max}$ is indicative of sprinting performance – faster top speeds are achieved by applying increased vertical forces [25]. Weyand et al. suggested that faster runners had briefer contact times which made greater vertical forces possible [25]. The simulation in this present work however suggests that greater $F_{max}$ was achieved for UB who actually had a longer contact time and ran at a faster velocity than his competitors. This present study and others [i.e., 14, 22, and 25] have all focused on the vertical force component. However it should be noted that anterior-posterior forces are also manipulated to improve sprint performance [18,19].

The estimated peak displacement of the leg spring ($\Delta L$) was comparable to that reported previously using the same model [22]. The maximal downward displacement of the CoM was markedly reduced in these elite sprinters compared to novice sprinters [22] indicating at maximum speed elite sprinters exhibit reduced vertical displacement. This along with the markedly increased $F_{max}$ (which is comparable (~3.0kN) to sprinters running at 10.37 m s$^{-1}$[6]) resulted in an increase in $K_{vert}$ compared to slower sprinters. As speed increases $K_{vert}$ has also been shown to increase [10]. The $K_{vert}$ reported here is 3.8–5.7 times greater than for slower sprinters [22]. These results also show that even though UB achieved the greatest velocity his $K_{vert}$ was significantly lower than his competitors. This paradox is partly due to the significantly increased $t_t$ for UB compared to TG and AP. Similarly $t_t$ along with significantly greater $\Delta L$ for UB resulted in a significantly lower $K_{leg}$ compared to TG and AP.

The data suggest increased $t_t$ for UB results in a decreased $K_{vert}$ and $K_{leg}$ compared to his competitors. This agrees with Morin et al. who manipulated $t_t$ at running speed of 3.33 m$^{-1}$ and showed that $K_{vert}$ and $K_{leg}$ decreased when $t_t$ was increased and when $t_t$ was decreased $K_{vert}$ and $K_{leg}$ increased [23]. Arampatzis et al. [2] showed that a reduction in $t_t$ during drop jumps resulted in an increase in stiffness. Furthermore Arampatzis et al. [2] reported an increase in maximum vertical ground reaction force.


This document was downloaded for personal use only. Unauthorized distribution is strictly prohibited.
and a greater vertical displacement of the center of mass when \( t_c \) was increased agreeing with the results for the sprinters in this current study.

As velocity increases from slow (2.0 m s\(^{-1}\)) to moderate (8.24 m s\(^{-1}\)) K\(_{leg}\) also increases by 60\% (11.2 kN m\(^{-1}\)–17.0 kN m\(^{-1}\)) [7]. For the 3 sprinters in this present study, UB K\(_{leg}\) was comparable to sprinters running at slower velocities [15,22] however K\(_{leg}\) for TG and AP were 1.6 and 1.5 times greater, respectively, compared to slower sprinters reported in the literature [15,22]. The greater vertical and leg stiffness seen for TG and AP may help resist the collapse of the body during contact and enhance force production during push-off, ultimately resulting in increased step frequency [7]. For UB step frequency is less than his competitors and the lower K\(_{leg}\) and K\(_{spine}\) and increased \( \Delta L \) and \( \Delta y \) suggest greater compliance for UB thereby facilitating the storage and utilization of elastic energy during the stretch shortening cycle [7]. The greater \( t_c \) and lower step frequency for world class athletes whilst in competition. Even though UB in this present study the SMM characteristics were estimated for...

**Conclusion**

In this present study the SMM characteristics were estimated for world class athletes whilst in competition. Even though UB achieved the greatest velocity (12.3 m s\(^{-1}\)) over the 60–80 m split, compared to his competitors, his K\(_{leg}\) and K\(_{spine}\) were significantly lower. This reduction in stiffness is a consequence of the increased contact time and lower step frequency. The opportunity to take direct measurements in a world championship final to calculate SMM characteristics is unlikely to happen; therefore these data provide a unique estimation of the SMM at the elite level of competition.

**References**


**Notice:**

This article was changed according to the following erratum on July 26th, 2012.

**Erratum**

The article contains an error in the Abstract.

Instead of:

This reduction in stiffness is a consequence of Bolt’s longer contact time (0.91 s) and lower step frequency (4.49 Hz). It should read....

This reduction in stiffness is a consequence of Bolt’s longer contact time (0.091 s) and lower step frequency (4.49 Hz).